

Stability of the Einstein static universe in IR modified Hořava gravity

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Recently, Hořava proposed a power counting renormalizable theory for (3+1)-dimensional quantum gravity, which reduces to Einstein gravity with a non-vanishing cosmological constant in IR, but possesses improved UV behaviors. In this work, we analyze the stability of the Einstein static universe by considering linear homogeneous perturbations in the context of an IR modification of Hořava gravity, which implies a ‘soft’ breaking of the ‘detailed balance’ condition. The stability regions of the Einstein static universe is parameterized by the linear equation of state parameter $w = p/\rho$ and the parameters appearing in the Hořava theory, and it is shown that a large class of stable solutions exists in the respective parameter space.

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I. INTRODUCTION

Recently, motivated by the Lifshitz model in condensed matter physics, Hořava proposed a power counting renormalizable theory for (3+1)-dimensional quantum gravity [1, 2]. This theory, denoted as Hořava-Lifshitz gravity, is believed to be the potential ultraviolet (UV) completion of general relativity (GR). In the infrared (IR) limit (setting the parameter $\lambda = 1$ in the action), it recovers GR. Hořava-Lifshitz gravity admits a Lifshitz scale-invariance in time and space, exhibiting a broken Lorentz symmetry at short scales, while at large distances higher derivative terms do not contribute, and the theory reduces to standard GR. Since then various properties and characteristics of the Hořava gravities have been extensively analyzed, ranging from formal developments [3], cosmology [4], dark energy [5, 6] and dark matter [7], spherically symmetric solutions [8, 9], and its viability with observational constraints [10] were also explored. Although a generic vacuum of the theory is the anti-de Sitter one, particular limits of the theory allow for the Minkowski vacuum. In this limit post-Newtonian coefficients coincide with those of pure GR. Thus, the deviations from conventional GR can be tested only beyond the post-Newtonian corrections, that is for a system with strong gravity at astrophysical scales.

In this work, we consider the stability of the Einstein static universe in Hořava-Lifshitz gravity, with a ‘soft’ violation of the detailed balance condition (Recently, the stability of the Einstein static universe in Hořava-Lifshitz gravity satisfying the detailed balance condition was analyzed [11]). The presence of the respective term in the action which represents a ‘soft’ violation of the ‘detailed balance’ condition modifies the IR behavior. Note that this IR modification term, with an arbitrary cosmological constant, represent the analogs of the standard Schwarzschild-(A)dS solutions, which were absent in the original Hořava model. The analysis of the static Einstein Universe is motivated by the possibility that the universe might have started out in an asymptotically Einstein static state, in the inflationary universe context [12]. On the other hand, the Einstein cosmos has always been of great interest in various gravitational theories.

In GR for instance, generalizations with non-constant pressure have been analyzed in [13]. In brane world models, the Einstein static universe was investigated in [14], while its generalization within Einstein-Cartan theory can be found in [15], and in loop quantum cosmology, we refer the reader to [16]. In the context of $f(R)$ modified theories of gravity, the stability of the Einstein static universe was also analyzed by considering homogeneous perturbations [17]. By considering specific forms of $f(R)$, the stability regions of the solutions were parameterized by a linear equation of state parameter $w = p/\rho$. Contrary to classical GR, it was found that in $f(R)$ gravity a stable Einstein cosmos with a positive cosmological constant does indeed exist. Thus, in principle, modifications in $f(R)$ gravity stabilize

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solutions which are unstable in GR. Furthermore, in [18] it was found that only one class of $f(R)$ theories admits an Einstein static model, and that this class is neutrally stable with respect to vector and tensor perturbations for all equations of state on all scales. These results are apparently contradictory with those of Ref. [17]. However, in a recent work, homogeneous and inhomogeneous scalar perturbations in the Einstein static solutions were analyzed [19], consequently reconciling both of the above works. In the context of modified theories of gravity, the stability of the Einstein static universe in $f(G)$ Gauss-Bonnet modified gravity was also analyzed [20]. In particular, by considering a generic form of $f(G)$, the stability regions of the Einstein static universe were parameterized by the linear equation of state and the second derivative $f''(G)$ of the Gauss-Bonnet term. It was shown that stable modes for all equation of state parameters w exist, if the parameters of the theory are chosen appropriately. Thus, the results show that perturbation theory of modified theories of gravity present a richer stability/instability structure than in GR.

Thus, it is the purpose of the present paper to consider the stability of the Einstein static universe by considering linear homogeneous perturbations in Hořava-Lifshitz gravity. Indeed, this analysis is particularly important as the higher derivative terms in the action contributes with a $1/a^4$ term in the modified Friedman equations. This contribution becomes dominant for small a , and as mentioned above motivates this analysis, due to the possibility that the universe might have started out in an asymptotically Einstein static state, in the inflationary universe context [12]. On the other hand, the cosmological solutions of GR are recovered at large scales. It is shown that a large class of Einstein static universes exist that are stable with respect to linear homogeneous perturbations.

This paper is outlined in the following manner: In Sec. II, we briefly review the action and field equations of Hořava gravity, and the respective modified Friedman equations. In Sec. III, we consider linear homogeneous perturbations in the context of the Einstein static Universe in Hořava gravity, and analyze the respective stability regions. In Sec. IV, we conclude.

II. HOŘAVA GRAVITY AND FIELD EQUATIONS

A. Action

Using the ADM formalism, the four-dimensional metric is parameterized by the following

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (1)$$

where N is the lapse function, N^i is the shift vector, and g_{ij} is the 3-dimensional spatial metric.

In this context, the Einstein-Hilbert action is given by

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{g} N (K_{ij} K^{ij} - K^2 + R^{(3)} - 2\Lambda), \quad (2)$$

where G is Newton's constant, $R^{(3)}$ is the three-dimensional curvature scalar for g_{ij} , and K_{ij} is the extrinsic curvature defined as

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad (3)$$

where the overdot denotes a derivative with respect to t , and ∇_i is the covariant derivative with respect to the spatial metric g_{ij} .

Consider the IR-modified Hořava action given by

$$S = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2\nu^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2\nu^2} \epsilon^{ijk} R_{il}^{(3)} \nabla_j R^{(3)l}_k - \frac{\kappa^2 \mu^2}{8} R_{ij}^{(3)} R^{(3)ij} + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{4\lambda - 1}{4} (R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2 \right) + \frac{\kappa^2 \mu^2 \varpi}{8(3\lambda - 1)} R^{(3)} \right], \quad (4)$$

where κ , λ , ν , μ , ϖ and Λ_W are constant parameters. C^{ij} is the Cotton tensor, defined as

$$C^{ij} = \epsilon^{ikl} \nabla_k \left(R^{(3)j}_l - \frac{1}{4} R^{(3)} \delta^j_l \right). \quad (5)$$

Note that the last term in Eq. (4) represents a 'soft' violation of the 'detailed balance' condition, which modifies the IR behavior. This IR modification term, $\mu^4 R^{(3)}$, generalizes the original Hořava model (we have used the notation of

Ref. [9]). Note that now these solutions with an arbitrary cosmological constant represent the analogs of the standard Schwarzschild-(A)dS solutions, which were absent in the original Hořava model [9].

The fundamental constants of the speed of light c , Newton's constant G , and the cosmological constant $\bar{\Lambda}$ are defined as

$$c^2 = \frac{\kappa^2 \mu^2 |\Lambda_W|}{8(3\lambda - 1)^2}, \quad G = \frac{\kappa^2 c^2}{16\pi(3\lambda - 1)}, \quad \bar{\Lambda} = \frac{3}{2} \Lambda_W c^2. \quad (6)$$

B. Modified Friedman equations

Consider the homogeneous and isotropic cosmological solution given by the following metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (7)$$

where $k = +1, 0, -1$ corresponds to a closed, flat, and open universe, respectively.

We assume that the matter contribution takes the form of a perfect fluid, with ρ and p the energy density and the pressure, respectively, so that the modified Friedman equations in Hořava gravity take the following form [6]

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{\kappa^2}{6(3\lambda - 1)} \left[\rho + \epsilon \frac{3\kappa^2 \mu^2}{8(3\lambda - 1)} \left(-\frac{k^2}{a^4} + \frac{2k(\Lambda_W - \varpi)}{a^2} - \Lambda_W^2 \right) \right], \quad (8)$$

$$\frac{\ddot{a}}{a} = \frac{\kappa^2}{6(3\lambda - 1)} \left[-\frac{1}{2}(\rho + 3p) + \epsilon \frac{3\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{k^2}{a^4} - \Lambda_W^2 \right) \right], \quad (9)$$

where $\epsilon = \pm 1$.

The analytic continuation $\mu^2 \rightarrow -\mu^2$ for the dS case, i.e., $\Lambda_W > 0$, is considered [6], and the upper (lower) sign denotes the AdS (dS) case. It is interesting to note that the higher derivative term appearing in the action (4) contributes with a $1/a^4$ term, and only exists for $k \neq 0$. This term dominates for low values of a , and the general relativistic cosmological solutions are recovered for large scales.

III. THE EINSTEIN STATIC UNIVERSE IN HOŘAVA GRAVITY AND PERTURBATIONS

A. Field equations

For the Einstein static universe, $a = a_0 = \text{const}$ and $k = 1$, the Ricci scalar becomes $R = 6/a_0^2$, (note that for $\lambda = 1$, GR is obtained in the IR limit). Furthermore, we consider a linear equation of state, $p = w\rho$, so that the field equations in this case are expressed in the following manner

$$\rho_0 = \frac{\epsilon \kappa^2 \mu^2}{a_0^4} \frac{[2 + 3a_0^2(1 + w)\varpi \pm \sqrt{4 - 6a_0^2(1 + w)(1 + 3w)\varpi}]}{6(1 + w)^2(3\lambda - 1)}, \quad (10)$$

$$\Lambda_W a_0^2 = \frac{(1 + 3w) \mp \sqrt{4 - 6a_0^2(1 + w)(1 + 3w)\varpi}}{3(1 + w)}, \quad (11)$$

where ρ_0 and p_0 are the unperturbed energy density and isotropic pressure, respectively. Note that it is useful to introduce the dimensionless parameters $\Omega := a_0^2 \varpi$ and $\Lambda = \Lambda_W a_0^2$. These relationships are useful to be written in this form, as one has a first glance at the existence issue that ρ_0 and Λ should be real, which imposes the following condition

$$2 - 3(1 + w)(1 + 3w)\Omega \geq 0. \quad (12)$$

The above inequality needs to be analyzed for the three cases $w < -1$, $-1 < w < -1/3$ and $w > -1/3$ which places restrictions on the allowed values of Ω . However, as we are primarily interested in a physically reasonable Einstein static universe, we will furthermore require positivity of the energy density, $\rho_0 > 0$. To further simplify the subsequent analysis, we will also assume $\lambda > 1/3$.

These useful conditions imply the following existence conditions for the upper sign of Eq. (10):

$$\begin{aligned}
\epsilon = +1, \quad w < -1, \quad \Omega &< \frac{2}{3} \frac{1}{(1+w)(1+3w)}, \\
\epsilon = +1, \quad -1 < w < -2/3, \quad \Omega &> \frac{2}{3} \frac{1}{(1+w)(1+3w)}, \\
\epsilon = +1, \quad -2/3 < w < -1/3, \quad \Omega &> -2, \\
\epsilon = +1, \quad -1/3 < w, \quad -2 < \Omega &< \frac{2}{3} \frac{1}{(1+w)(1+3w)}, \\
\epsilon = -1, \quad -2/3 < w < -1/3, \quad \frac{2}{3} \frac{1}{(1+w)(1+3w)} &< \Omega < -2, \\
\epsilon = -1, \quad -1/3 < w, \quad \Omega &< -2,
\end{aligned} \tag{13}$$

which are depicted in the left plots of Figs. 1 and 2, respectively.

For the lower sign, of Eq. (10), we find:

$$\begin{aligned}
\epsilon = +1, \quad w < -1, \quad \Omega &< -2, \\
\epsilon = +1, \quad w < -1, \quad 0 < \Omega &< \frac{2}{3} \frac{1}{(1+w)(1+3w)}, \\
\epsilon = +1, \quad -1 < w < -2/3, \quad \frac{2}{3} \frac{1}{(1+w)(1+3w)} &< \Omega < -2, \\
\epsilon = +1, \quad -1 < w < -1/3, \quad 0 < \Omega, \\
\epsilon = +1, \quad -1/3 < w, \quad 0 < \Omega &< \frac{2}{3} \frac{1}{(1+w)(1+3w)}, \\
\epsilon = -1, \quad w < -2/3, \quad -2 < \Omega &< 0, \\
\epsilon = -1, \quad -2/3 < w < -1/3, \quad \frac{2}{3} \frac{1}{(1+w)(1+3w)} &< \Omega < 0, \\
\epsilon = -1, \quad -1/3 < w, \quad \Omega &< 0.
\end{aligned} \tag{14}$$

which are depicted in the right plots of Figs. 1 and 2, respectively.

The above inequalities with the upper and lower signs, and with $\epsilon = +1$, are represented graphically in Fig. 1, and with $\epsilon = -1$ in Fig. 2, respectively. Note that there are many configurations which allow for an Einstein static universe. This is in contrast with other modifications of GR, such as $f(R)$ modified gravity, where there exists a unique background Einstein static universe for every choice of $f(R)$.

B. Linear homogeneous perturbations

In what follows, we analyze the stability against linear homogeneous perturbations around the Einstein static universe given in Eqs. (8)-(9). Thus, we introduce perturbations in the energy density and the metric scale factor which only depend on time

$$\rho(t) = \rho_0 + \delta\rho_1(t), \quad a(t) = a_0 + \delta a_1(t). \tag{15}$$

Now we consider adiabatic perturbations which also satisfy a linear equation of state, $\delta p(t) = w\delta\rho(t)$ and linearize the perturbed field equations. Firstly, we consider Eq. (8) which upon subtracting the background field equation yields

$$\delta\rho_1 = \frac{3\epsilon\kappa^2\mu^2}{2a_0^5(3\lambda-1)}(\Lambda - \Omega - 1)\delta a_1. \tag{16}$$

Next, we perturb the evolution equation (9) and eliminate the perturbed energy density by virtue of the latter equation. The resulting second order differential equation for δa_1 is given by

$$\delta a_1''(t) + \frac{\epsilon\kappa^4\mu^2}{8a_0^4(3\lambda-1)^2}[(1 + \Lambda - \Omega) - 3w(1 - \Lambda + \Omega)]\delta a_1(t) = 0. \tag{17}$$

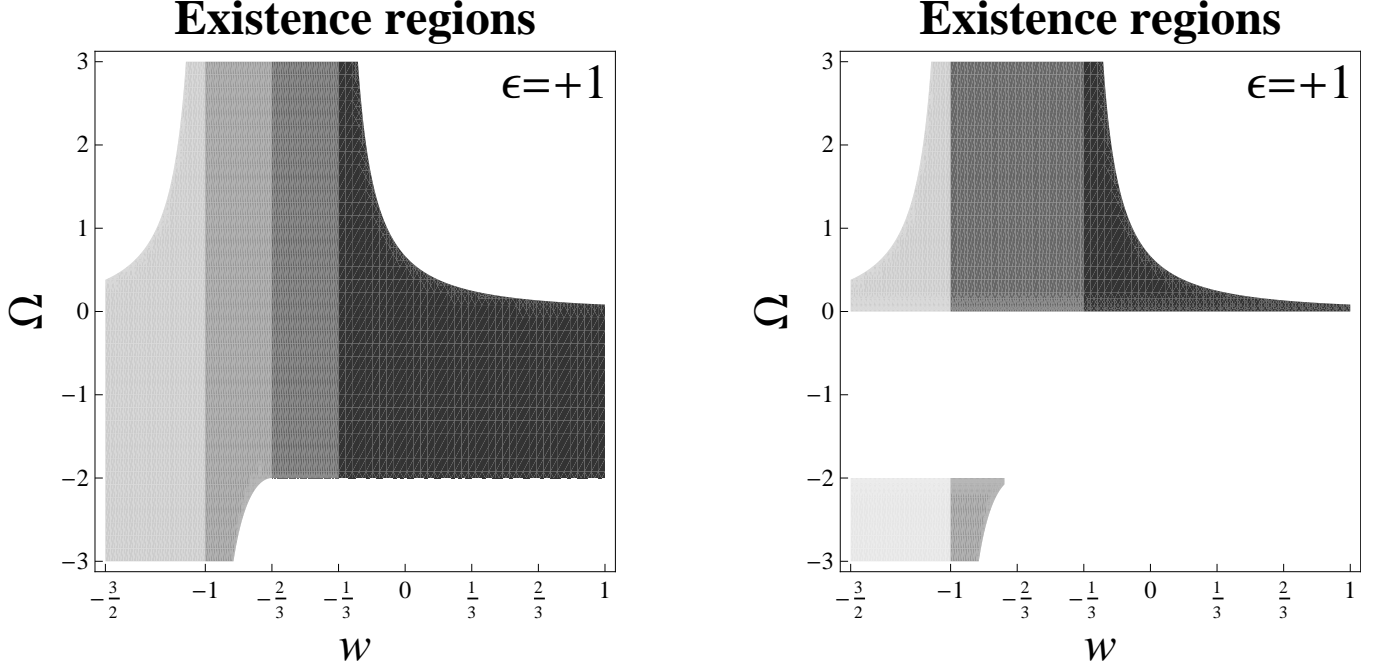


FIG. 1: Regions of existence in the (w, Ω) parameter space for the specific case of $\epsilon = +1$. The left panel shows the existence regions for the upper sign of Eq. (10), while the right panel is for the lower sign. The different shades of gray correspond to the different inequalities.

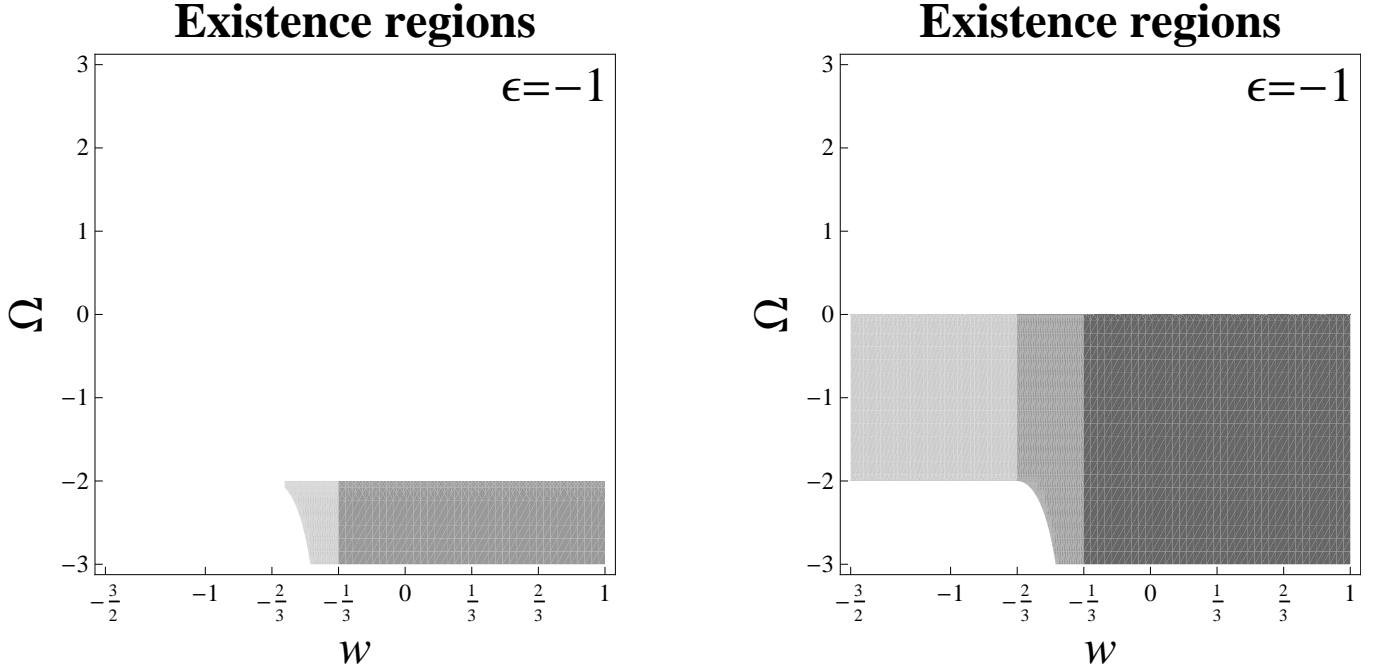


FIG. 2: Regions of existence in the (w, Ω) parameter space for the specific case of $\epsilon = -1$. The left panel shows the existence regions for the upper sign of Eq. (10), while the right panel is for the lower sign. The different shades of gray correspond to the different inequalities.

As only the sign of the prefactor of the second term is relevant, we can rescale $\delta a_1(t)$ appropriately and consider

$$\delta a_1''(t) + \epsilon[(1 + \Lambda - \Omega) + 3w(-1 + \Lambda - \Omega)]\delta a_1(t) = 0. \quad (18)$$

Before solving this equation it should be noted that Λ is determined by the background, see Eq. (11) and hence this quantity should also be substituted. We find

$$\Lambda - \Omega = \frac{(1 - 3\Omega) + 3w(1 - \Omega) \mp \sqrt{4 - 6(1 + w)(1 + 3w)\Omega}}{3(1 + w)}. \quad (19)$$

Using the standard ansatz $a_1(t) = A \exp(iWt)$, where A and W are constants, we find that the above differential equation (18) provides the following solutions

$$W = \pm \sqrt{\epsilon} \sqrt{(1 + \Lambda - \Omega) + 3w(-1 + \Lambda - \Omega)}. \quad (20)$$

C. Stability regions

To analyze the stability regions, first consider the cases $\epsilon = +1$ ($\epsilon = -1$). Now, the stability conditions impose that the factor within the square root of Eq. (20) is positive (negative), which is translated by the following inequalities:

$$\epsilon = +1 \quad (1 + \Lambda - \Omega) + 3w(-1 + \Lambda - \Omega) > 0, \quad (21)$$

$$\epsilon = -1 \quad (1 + \Lambda - \Omega) + 3w(-1 + \Lambda - \Omega) < 0. \quad (22)$$

Since Λ is determined by the background equations, see (19), the equations become more complicated. For the upper sign we have:

$$\begin{aligned} \epsilon = +1, \quad -1 < w < -1/3, \quad \Omega &> \frac{2}{3} \frac{1}{(1 + w)(1 + 3w)}, \\ \epsilon = +1, \quad -1/3 < w < 1/3, \quad \Omega &< \frac{2}{3} \frac{1}{(1 + w)(1 + 3w)}, \\ \epsilon = +1, \quad 1/3 < w, \quad \Omega &< \frac{1 - 3w}{1 + 3w} - \frac{1}{\sqrt{3}} \sqrt{\frac{-1 + 3w}{1 + w}}, \\ \epsilon = +1, \quad 1/3 < w, \quad \frac{1 - 3w}{1 + 3w} + \frac{1}{\sqrt{3}} \sqrt{\frac{-1 + 3w}{1 + w}} &< \Omega < \frac{2}{3} \frac{1}{(1 + w)(1 + 3w)}, \\ \epsilon = -1, \quad w < -1, \quad \Omega &< \frac{2}{3} \frac{1}{(1 + w)(1 + 3w)}, \\ \epsilon = -1, \quad 1/3 < w, \quad \frac{1 - 3w}{1 + 3w} - \frac{1}{\sqrt{3}} \sqrt{\frac{-1 + 3w}{1 + w}} &< \Omega < \frac{1 - 3w}{1 + 3w} + \frac{1}{\sqrt{3}} \sqrt{\frac{-1 + 3w}{1 + w}}, \end{aligned} \quad (23)$$

which are depicted in the left plots of Figs. 3 and 4, respectively.

Finally, for the lower sign we obtain:

$$\begin{aligned} \epsilon = +1, \quad w < -1, \quad \frac{1 - 3w}{1 + 3w} - \frac{1}{\sqrt{3}} \sqrt{\frac{-1 + 3w}{1 + w}} &< \Omega < \frac{1 - 3w}{1 + 3w} + \frac{1}{\sqrt{3}} \sqrt{\frac{-1 + 3w}{1 + w}}, \\ \epsilon = +1, \quad -1 < w < -1/3, \quad \Omega &> \frac{2}{3} \frac{1}{(1 + w)(1 + 3w)}, \\ \epsilon = +1, \quad -1/3 < w, \quad \Omega &< \frac{2}{3} \frac{1}{(1 + w)(1 + 3w)}, \\ \epsilon = -1, \quad w < -1, \quad \Omega &< \frac{1 - 3w}{1 + 3w} - \frac{1}{\sqrt{3}} \sqrt{\frac{-1 + 3w}{1 + w}}, \\ \epsilon = -1, \quad w < -1, \quad \frac{1 - 3w}{1 + 3w} + \frac{1}{\sqrt{3}} \sqrt{\frac{-1 + 3w}{1 + w}} &< \Omega < \frac{2}{3} \frac{1}{(1 + w)(1 + 3w)}, \end{aligned} \quad (24)$$

which are depicted in the right plots of Figs. 3 and 4, respectively.

By combining the existence conditions with the stability conditions, we can identify a large class of Einstein static universes which are stable with respect to homogeneous perturbations. It should also be noted that for the upper sign a Einstein static universe of phantom matter ($w < -1$) exists. Superimposing the inequality plots Figs. 1–4 we can picture the complete parameter space for which the Einstein static universe in IR modified Hořava gravity exists and

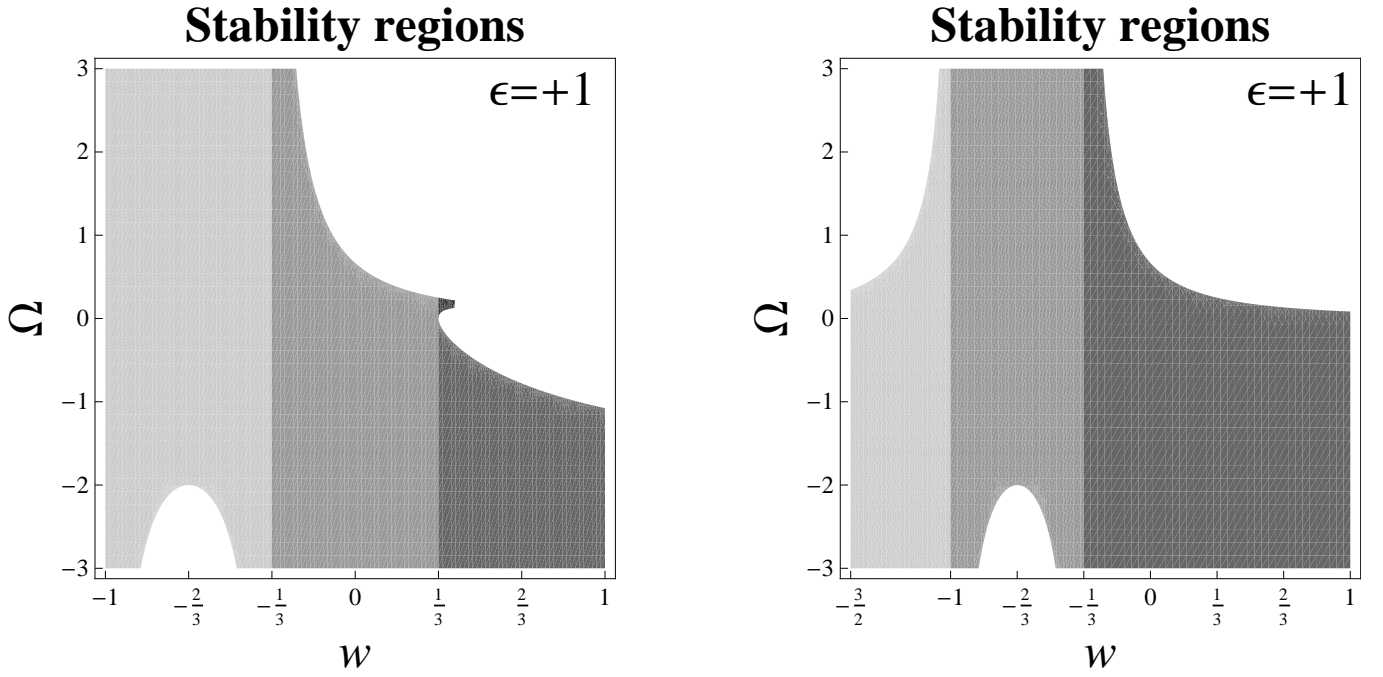


FIG. 3: Regions of stability in the (w, Ω) parameter space (left panel, upper sign; right panel, lower sign), for the specific case of $\epsilon = +1$. See the text for details.

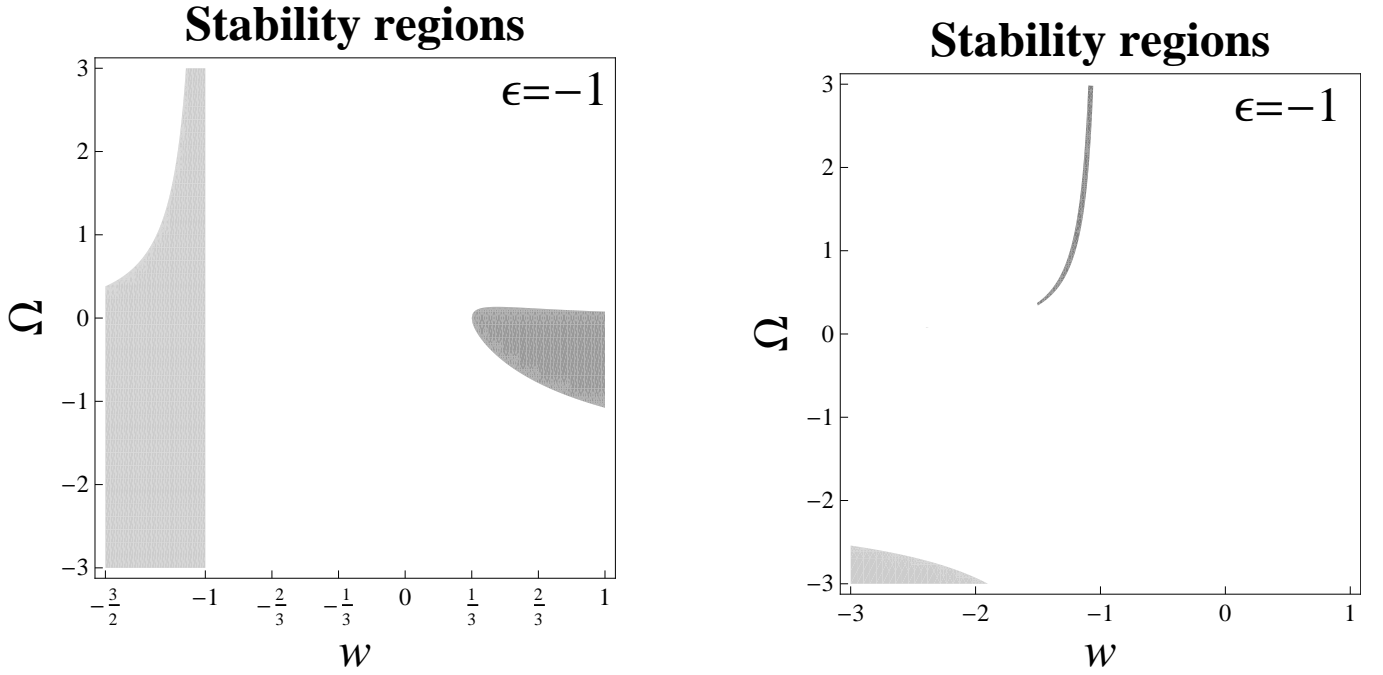


FIG. 4: Regions of stability in the (w, Ω) parameter space (left panel, upper sign; right panel, lower sign), for the specific case of $\epsilon = -1$. See the text for details.

is stable with respect to linear homogeneous perturbations. This is depicted in Fig. 5 for the specific case of $\epsilon = +1$. For the specific case of $\epsilon = -1$, we verify the nonexistence of the Einstein static universe existence/stability regions.

For every parameter choice we can compute the actual value of Λ by using Eq. (11). Note, however, that it is very involved to represent the values Λ can attain in general because this would require to plot a surface in the (w, Ω, Λ) parameter space which satisfies various inequalities simultaneously.

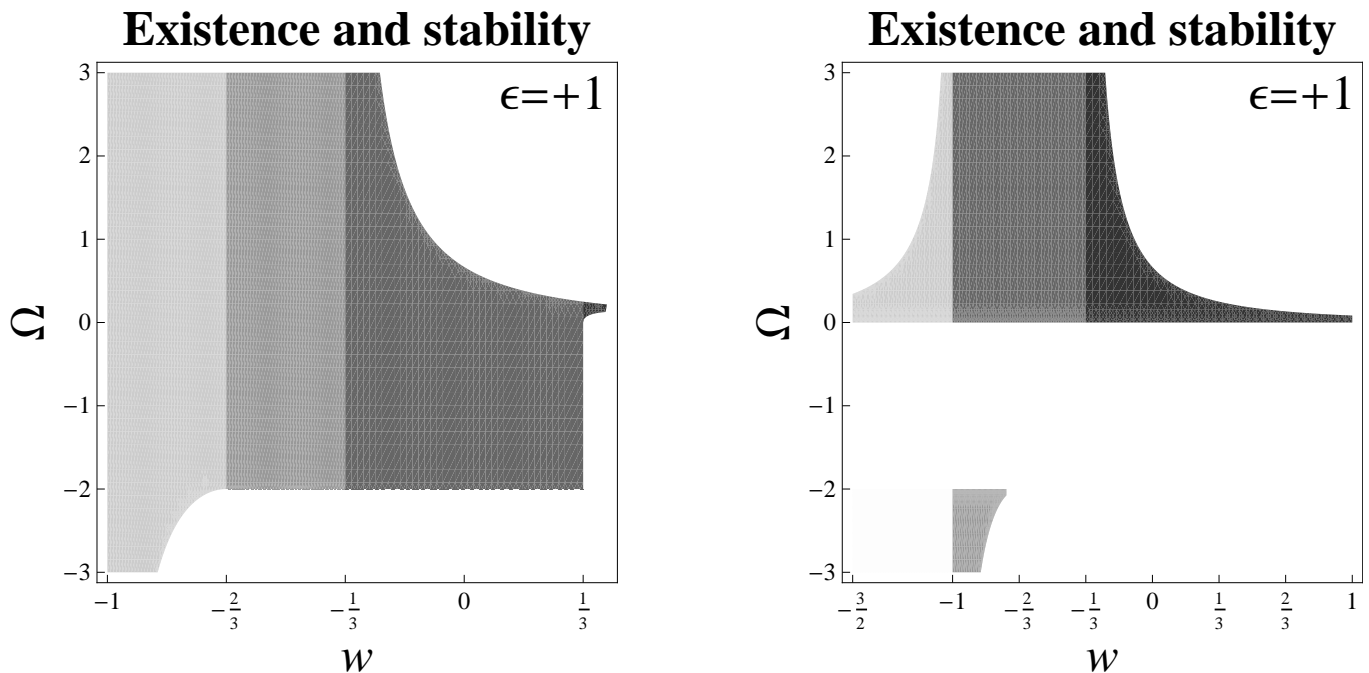


FIG. 5: Combined region of existence and stability in the (w, Ω) parameter space (left panel, upper sign; right panel, lower sign) for the specific case of $\epsilon = +1$.

IV. SUMMARY AND DISCUSSION

The Einstein static universe has recently been revived as the asymptotic origin of an emergent universe, namely, as an inflationary cosmology without a singularity [12]. The role of positive curvature, negligible at late times, is crucial in the early universe, as it allows these cosmologies to inflate and later reheat to a hot big-bang epoch. An attractive feature of these cosmological models is the absence of a singularity, of an ‘initial time’, of the horizon problem, and the quantum regime can even be avoided. Furthermore, the Einstein static universe was found to be neutrally stable against inhomogeneous linear vector and tensor perturbations, and against scalar density perturbations provided that the speed of sound satisfies $c_s^2 > 1/5$ [23]. Further issues related to the stability of the Einstein static universe may be found in Ref. [24].

In this work we have analyzed linear homogeneous perturbations around the Einstein static universe in the context of IR modified Hořava gravity. In particular, perturbations in the energy density and the metric scale factor were introduced, a linear equation of state, $p(t) = w\rho(t)$, was considered, and finally the linearized perturbed field equations and the dynamics of the solutions were analyzed. It was shown that stable modes for all equation of state parameters w exist and, in particular, the complete parameter space for which the Einstein static universe in IR modified Hořava gravity exists and is stable with respect to linear homogeneous perturbations was presented. Thus, as in Refs. [17, 18, 19, 20] our results show that perturbation theory of modified theories of gravity present a richer stability/instability structure than in general relativity. Finally, it is of interest to extend our results to inhomogeneous perturbations in the spirit of Ref. [19], and to include the canonical scalar field case. Work along these lines is presently underway.

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